Worcester County Mathematics League

Freshman Meet 1 – November 1, 2006

Round 1: Evaluation of Algebraic Expressions and Order of Operations

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

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1. Following the proper order of operations, evaluate:

$$\left(\left((1-2)^2+3\right)^2-4\right)^2$$

2. If $a * b = \frac{a}{b}+\frac{b}{a}$ and $a \wedge b = \frac{ab}{a+b}$, evaluate $\left[\left(3 * 2\right)-2\right]^2 \cdot \left(3 \wedge 2\right)$.

3. If $A=-5\frac{1}{3}$, $B=8\frac{1}{4}$, and $C=-12$ find the simplified value of

$$\frac{\left[\left(A+B\right)C-\left(AB-C\right)\right]^2}{C-B}.$$

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

Hudson, Assabet Valley, Worcester Acad.

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Round 2: Solving Linear Equations

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

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1. Solve for x: $5x - 3(7 - x) = 2(x + 9)$

2. Solve for x: $\dfrac{4-3x}{3} - \dfrac{2x-5}{5} = -\left(\dfrac{\frac{2}{5}+8x}{6}\right)$

3. Solve for x: $5x + \dfrac{1}{3}\left(4x - 8\right) = \dfrac{3}{4}(9x - 4) - 3$

ANSWERS

(1 pt.) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

Worc. Academy, Bancroft, Auburn

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Round 3: Logic Problems

All answers must be in simplest exact form

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1. On a balance scale, one brick balances with three-quarters of a pound and three-quarters of a brick. In pounds, what is the weight of a single brick?

2. Five students are sitting in a room containing 3 stools and 2 chairs. Roy and Sally sit on the same type of seat. Sally and Randy sit on different types of seats. Randy and Jim also sit on different types of seats. The fifth student's name is Mary. Name the two students sitting on the chairs.

3. What is the units' digit in the decimal expansion of 2007^{2007}?

ANSWERS

(1 pt.) 1._____ pounds

(2 pts.) 2._____ and _____

(3 pts.) 3._____

Westborough, Notre Dame, QSC

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Round 4: Ratio, Proportion and Variation

All answers must be in simplest exact form

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1. If you are traveling at 40 miles per hour, how many miles will you travel in 7 minutes?

2. Christopher is standing 3 feet from a lamppost that is 12 feet tall. If his shadow created by a light atop the lamppost is $2\frac{7}{13}$ feet long, find Christopher's height <u>in feet</u>.

3. The "enjoyment index" for a movie varies directly as the square of the number of screens at the theater and inversely as the outside temperature. If the "enjoyment index" for a movie at a three-screen theater on a 63-degree day is 6, what is the "enjoyment index" for a movie showing at a 4-screen theater on an 84-degree day?

<u>ANSWERS</u>

(1 pt.) 1._____ miles

(2 pts.) 2._____ feet

(3 pts.) 3._____

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TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places (3 pts. each)

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1. Solve for p: $5p - 3\left[7 - (1 - 2p)\right] = -3(1 - p)$

2. Two consecutive positive integers are called a "hot pair" if their sum is prime. How many "hot pairs" are there if only the first 20 positive integers are used?

3. Two fishermen were discussing the day's catch. Russ said, "If you give me two of your fish, then I will have the same number as you will have." "I have a better idea," said Matt, "If you give me two of your fish, then I will have twice as many as you will have." Find the total number of fish that Russ and Matt had together.

4. For positive integers x and y, $x * y = x^y + y^x$. If $2 * a = 100$, find a.

5. One hundred and seventy-five percent of an integer K is an integer between 80 and 90. Find the value of K.

6. How many positive integer factors of 8100 are multiples of 12?

7. Points A, B, C and D are collinear (with B between A and C, and C between B and D). If $\frac{AB}{BD} = \frac{2}{5}$ and $\frac{AC}{CD} = \frac{11}{10}$, find the ratio $\frac{AB}{CD}$ as a simplified fraction.

8. The length of a rectangle is 1 centimeter longer than twice its width. The area of the rectangle is $45\ \text{cm}^2$. Find the dimensions of the rectangle in centimeters.

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ANSWER SHEET – TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places (3 pts. each)

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____ cm by _____ cm

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ANSWERS

Round 1

1. 144

2. $\frac{1}{30} = 0.0\overline{3}$

3. $-\frac{4}{9} = -0.\overline{4}$

Round 2

1. $6\frac{1}{2} = 6.5 = \frac{13}{2}$

2. 36

3. 8

Round 3

1. 3

2. Mary and Randy (any order)

3. 3

Round 4

1. $4\frac{2}{3} = 4.\overline{6}$

2. $5\frac{1}{2} = 5.5 = \frac{11}{2}$

3. 8

Team Round

1. $-\frac{15}{4} = -3.75 = -3\frac{3}{4}$

2. 11

3. 24

4. 6

5. 48

6. 12

7. $\frac{3}{5}$

8. 10 by 4.5 (or 4.5 by 10)

(4.5 can be written as $\frac{9}{2}$ or $4\frac{1}{2}$)

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BRIEF SOLUTIONS

Round 1

1. $\left(\left(\left(1-2\right)^2+3\right)^2-4\right)^2 = ((1+3)^2-4)^2 = 12^2 = 144$

2. Note that $a*b = \frac{a}{b}+\frac{b}{a} = \frac{a^2+b^2}{ab}$. So, $\left[\left(3*2\right)-2\right]^2 \cdot \left(3\wedge 2\right) = \left(\frac{13}{6}-2\right)^2 \cdot \left(\frac{6}{5}\right) = \frac{1}{36}\cdot\frac{6}{5} = \frac{1}{30}$

3. $\frac{\left[\left(A+B\right)C-\left(AB-C\right)\right]^2}{C-B} = \frac{(AC+BC-AB+C)^2}{C-B} = \frac{\left(\frac{-16}{3}\cdot(-4)+\frac{33}{4}(-12)+\frac{16}{3}\cdot\frac{33}{4}+(-12)\right)^2}{-12-\frac{33}{4}}$

$= \frac{(64-99+44-12)^2}{\frac{-81}{4}} = \frac{9}{\frac{-81}{4}} = -\frac{4}{9}$

Round 2

1. $5x-3(7-x) = 2(x+9) \Rightarrow 8x-21 = 2x+18 \Rightarrow 6x = 39 \Rightarrow x = \frac{13}{2}$

2. Multiply both sides by 30: $\frac{4-3x}{3}-\frac{2x-5}{5} = -\left(\frac{\frac{2}{5}-8x}{6}\right) \Rightarrow 40-30x-12x+30 = -2-40x \Rightarrow -2x = -72 \Rightarrow x = 36$

3. Multiply both sides by 12: $5x+\frac{1}{3}\left(4x-8\right) = \frac{3}{4}(9x-4)-3 \Rightarrow 60x+16x-32 = 81x-36-36 \Rightarrow 5x = 40 \Rightarrow x = 8$

Round 3

1. Let B = the weight of one brick and P = one pound. Then, $B = \frac{3}{4}P+\frac{3}{4}B \Rightarrow \frac{1}{4}B = \frac{3}{4}P \Rightarrow B = 3P$.

2. Since Sally and Randy sit on different types of seats and Randy and Jim sit on different types of seats, Sally and Jim must sit on the same type of seat. Also, Sally sits on the same kind of seat as Roy. Hence, Sally, Roy and Jim must sit on the three available stools, leaving the chairs for Mary and Randy.

3. Explores powers of 7: $7^1 = 1,\ 7^2 = 49,\ 7^3 = 343,\ 7^4 = 2401,\ 7^5 = 16807$. The powers of 7 cycle with a period of 4. Since $2007 \div 4 = 501$ remainder 3, 2007^{2007} has the same units digit as $7^3 = 343$. Hence, the units' digit is 3.

Round 4

1. Change the units of measure: $\frac{40 \text{ mi}}{\text{hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times 7 \text{ min} = \frac{14}{3} \text{ mi}$

2. Set up a proportion between Chris' height (call it *x*) and the height of the lamppost and the length of Chris' shadow and the total length from the lamppost to the tip of his shadow (in feet): $\frac{x}{12} = \frac{33/13}{72/13} \Rightarrow \frac{72}{13}x = 12 \cdot \frac{33}{13} \Rightarrow x = 5\frac{1}{2}$

3. Let *E* = the enjoyment index, *S* = the number of screens and *T* = the outside temperature. The model is $E = \frac{kS^2}{T}$, where *k* is the constant of proportionality. The first piece of information gives us $6 = \frac{9k}{63} \Rightarrow k = 42$. The second piece of information allows us to solve for *E*: $E = \frac{42 \cdot 16}{84} = 8$.

Team Round

1. $5p - 3\left[7 - (1 - 2p)\right] = -3(1 - p) \Rightarrow 5p - 18 - 6p = -3 + 3p \Rightarrow 15 = 4p \Rightarrow p = -\frac{15}{4}$

2. Notice that the sum of two consecutive positive integers (beginning with 1 + 2 = 3, and ending at 19 + 20 = 39) gives us the odd integers starting at 3 and ending at 39. The primes between 3 and 39, inclusive, are 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and 37. There are 11.

3. Let *R* = the number of fish Russ has, and *M* = the number of fish that Matt has. Then, $R + 2 = M - 2$ and $2(R - 2) = M + 2$. Solving simultaneously (by substituting $M = 2(R - 2) - 2$ into the first equation) gives $R + 2 = 2R - 4 - 2 - 2 \Rightarrow R = 10$ and *M* = 14. The total number of fish is 24.

4. Observe the value of $2^a + a^2$ for consecutive integer values of *a* starting with 1:

a	$2^a + a^2$
1	3
2	8
3	17
4	32
5	57
6	100

The answer is *a* = 6.

5. K is an integer multiple of 4, so we have $80 < \frac{7}{4}K < 90 \Rightarrow 320 < 7K < 360 \Rightarrow 45.7... < K < 51.4... \Rightarrow K = 48$

6. Observe that $8100 = 12(3^3 \cdot 5^2)$. Therefore there are $(3 + 1)(2 + 1) = 12$ different multiples of 12.

7. Let *AD* = 21, so *AB* = 6 and *BD* = 15. Therefore, *BC* = 5 and *CD* = 10. Hence, $\frac{AB}{CD} = \frac{6}{10} = \frac{3}{5}$.

8. Let *l* = length of the rectangle and *w* = the rectangle's width. Then,

$A = w(2w + 1) = 45 \Rightarrow 2w^2 + w - 45 = 0 \Rightarrow (2w - 9)(w + 5) = 0 \Rightarrow w = \frac{9}{2}$ and the rectangle's length is 10.